Determination of the Residual Stress-Crack Opening Relationship of SFRC Flexural Members

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Abstract. Steel fibre reinforced concrete (SFRC) has become widespread material in building areas such as underground shotcrete structures or industrial floors. However, due to the absence of universally accepted guidelines for SFRC, application fields of this material are still limited. This paper deals with assessment of the residual stresses of tensile SFRC. An adequate method for determination of residual stress-crack opening relation, based on test data of three-point bending beams is proposed. To verify the analysis results a numerical modelling is utilized employing a nonlinear finite element analysis program. Simulated load-crack width curves were compared with the experimental data validating adequacy of the proposed model.

Keywords: residual stresses, steel fibre reinforced concrete, flexural members, crack width.
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INTRODUCTION

Steel fibre reinforced concrete (SFRC) is a cement-based composite material with discrete, randomly distributed, fibres. Due to the ability of fibres to transfer stresses through the crack plane, cracked SFRC is able to carry a certain portion of tensile stresses. Post-cracking strength is the main parameter that describes behaviour of SFRC. Tensile stresses appeared in a cracked SFRC are known as residual. Because of great number of different types of fibres, quantification of the post-cracking strength provided by interaction of fibres with concrete becomes a rather complicated issue. Generally, SFRC can be considered as a concrete with randomly dispersed fibre reinforcement or as a homogeneous material – conventional concrete with improved material properties. While modelling the behaviour of each discrete fibre is a complex procedure, research carried out at a macro-scale, considering SFRC as a homogeneous material, is commonly preceded. Such analysis is often associated to inverse (constitutive) analysis of the standard [1-4] or alternative [5, 6] test results.

The research reported in this paper is dedicated to the determination of residual strength of SFRC in tension. The mechanically sound method for the inverse analysis of SFRC is proposed. Adequacy of the calculated results was validated using experimental data of standard flexural specimens. Computational effectiveness was proved employing the derived residual strength models into nonlinear finite element analysis program.

CONSTITUTIVE MODELLING OF SFRC

Most of the models found in literature propose simple and continuous non-differentiable constitutive diagrams that are characterized through macroscopic properties by means of inverse analysis procedures. These approaches focus either on residual stress-strain (σ_{fr}-ε) [1, 2] or residual stress-crack width (σ_{fr}-w) relationships [3, 4]. One of the ways to define the post-cracking behaviour of SFRC was proposed by Naaman [7]. A simple analytical expression for residual strength is suggested, which is able to account for pullout length ratio, the efficiency factor of fibre orientation in the cracked state:

$$f_r = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \tau \cdot V_f \cdot \beta \cdot \left(l_f / d_f \right), \tag{1}$$

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where λ_1 – expected pullout length ratio; λ_2 – efficiency factor of orientation in the cracked state; λ_3 – group reduction factor associated with the number of fibres pulling out per unit area; τ – average bond stress of a single fibre embedded in the concrete; V_f – steel fibre volume percentage; β – bond factor, in accordance to Campione [8], assumed (for hooked fibres) to be equal 1.0; l_f and d_f – length and diameter of fibre, respectively.

Another method for assessing tensile behaviour of cracked SFRC is proposed by RILEM TC 162-TDF [2]. The method uses a tri-linear stress – strain relation of SFRC in tension and is based on experimental data of the 3 point bending test performed on the specimens with dimensions of 150 x 150 x 600 mm. To control the cracking process a notch at mid-span of the beam is formed, which allows measuring the opening crack width. The post-cracking response of SFRC is characterized by residual stresses σ_{r1} and σ_{r2}, which are correspond to experimental load values at different beam deformation stages. On the basis of the experimental data, the residual stresses are determined by the following expressions:

$$\sigma_{r1} = 0.675kP_1L/\left(bh_{sp}^2\right); \quad \sigma_{r2} = 0.555kP_2L/\left(bh_{sp}^2\right). \qquad (2)$$

where k – scale factor; P_1 and P_2 – load recorded at the mid-span deflection equal to 0.46 mm and 3.0 mm or crack opening width equal to 0.5 mm and 3.5 mm; L – span; b – width of the specimen; h_{sp} – depth of the notched section.

EXPERIMENTAL PROGRAM

An experimental program was conducted in order to investigate the effect of steel fibres on the response of members subjected to bending. Several beams, with various fibre contents, were tested, containing 40 and 80 kg/m^3 hooked-ended steel fibres (respectively equivalent to 0.5% and 1.0% of the total specimen volume). Fibres with a length l_f of 50 mm and a diameter d_f of 1 mm, resulting in the aspect (length to diameter) ratio of 50 were used.

Standard specimens with dimensions of 150 × 150 × 600 mm and a span of 500 mm were casted according to Rilem TC162-TDF [3]. To localize cracking process, each beam was equipped with a notch of 25 mm at the mid-span. For experimental investigation of SFRC displacement controlled testing machine was used and beams were loaded with a rate of 0.2 mm/min. The mid-span deflections as well as crack width were measured within the test using linear variable displacement transducers (LVDT).

The compressive strength of concrete was measured at the age of 28 days on 150 mm cube specimens, resulting in average values of 43.1 MPa and 44.8 MPa for 0.5% and 1.0% steel fibre concrete mixes, respectively. From the test results of the individual beams in a test series, load-crack width curves have been constructed. Fig. 1 shows the load-crack width curves for the 0.5% and 1.0% of fibre volume mixes, respectively. It can be seen that, as the fibre volume increases, both the peak load and the post-peak load increase. However, the variability in the measured load is significant. Such a variety of results was mainly influenced by the orientation and number and of fibres acting at the crack section.

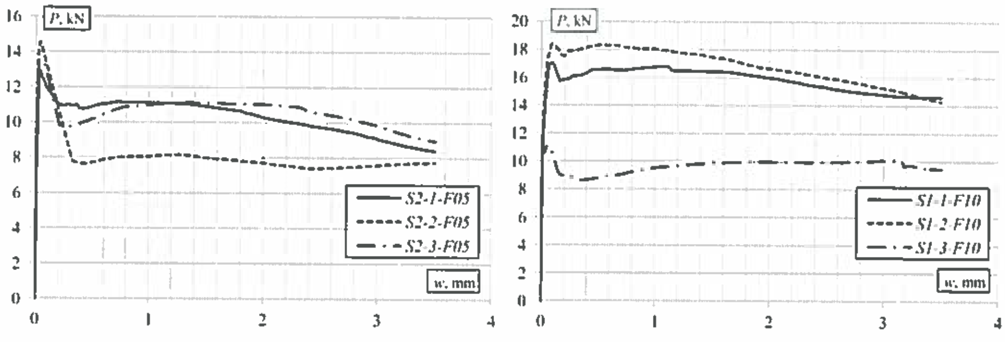

FIGURE 1. Experimental load-crack width relations for the SFRC beams width 0.5% (left) and 1.0% (right) fibres volume

PROPOSED TECHNIQUE FOR THE INVERSE ANALYSIS

The proposed inverse technique employs experimental data from standard three-point bending tests and uses classical principles of material mechanics. It is assumed that at the place of crack the tensile strain approaches infinity, whereas at a distance besides the crack, the strain is defined by the stress-strain relation before cracking. To determine tensile strain of SFRC at a certain loading step, a length, further called influence length, should be considered. The influence length over which the stresses are distributed due to the opening crack is taken equal to two times the height of the tensile zone [1]. The unknown position of the neutral axis is obtained from expression of residual stresses by simultaneously solving equilibrium equations of axial forces and bending moments:

$$\sigma_{r,n} = 0.5\varepsilon_n^2 E\left(h_{sp}/y_n - 1\right)^2 - 0.5 f_{ct}\varepsilon_{cr} - \sigma_{r,1}\left(\varepsilon_1 - \varepsilon_{cr}\right) - \sum\nolimits_{i=3}^{n} \sigma_{r,i-1}\left(\varepsilon_{i-1} - \varepsilon_{i-2}\right) =$$

$$= 2\left[\frac{P_n L \varepsilon_n^2}{4by_n^2} - \frac{\varepsilon_n^3 E}{3}\left(\frac{h_{sp}}{y_n} - 1\right)^3 - \frac{f_{ct}\varepsilon_{cr}^2}{3} - \frac{\sigma_{r,1}}{2}\left(\varepsilon_1 - \varepsilon_{cr}\right)\left(\varepsilon_{cr} + \varepsilon_1\right) - \sum_{i=3}^{n} \frac{\sigma_{r,i-1}}{2}\left(\varepsilon_{i-1} - \varepsilon_{i-2}\right)\left(\varepsilon_{i-2} + \varepsilon_{i-1}\right)\right] \Big/ \left(\varepsilon_{n-1} + \varepsilon_n\right). \tag{3}$$

Residual stresses for every loading step and, therefore, crack width, can be found from equilibrium of axial forces:

$$\varepsilon_n E \cdot \frac{\left(h_{sp} - y_n\right)^2}{2y_n} - \frac{f_{ct}^2 y_n}{2E\varepsilon_n} - y_n\sigma_1 \cdot \frac{\varepsilon_1 - \varepsilon_{cr}}{\varepsilon_n} - \frac{y_n}{\varepsilon_n} \cdot \sum_{i=2}^{n}\left(\varepsilon_i - \varepsilon_{i-1}\right)\sigma_{r,i} = 0; \quad \varepsilon_n = \frac{w_n}{2y_n}; \quad \varepsilon_{cr} = \frac{f_{ct}}{E}, \tag{4}$$

where n – loading steps; $\sigma_{r,i}$ – residual stresses at i-th step, ε_i – element deformation at i-th step; w_i – crack width at i-th step; y_i – position of neutral axis at i-th step; h_{sp} – depth of the notched section; E – Young's modulus of SFRC; f_{ct} – tensile strength of concrete; ε_{cr} – cracking strain of SFRC; P_i – loading at i-th step; L – span; b – width of the specimen. Obtained residual stress-crack opening width relations for the test beams are given in Fig. 2.

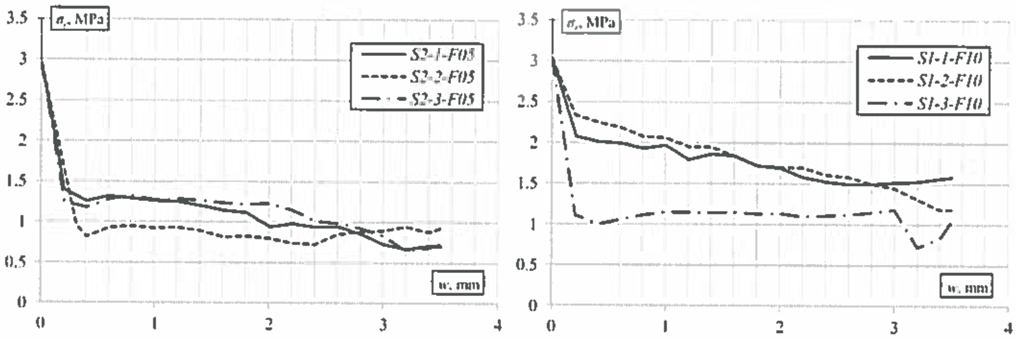

FIGURE 2. Determined residual stress-crack width relations for the beams with 0.5% (left) and 1.0% (right) fibres volume

NUMERICAL MODELLING

Adequacy of the residual stress-crack width relationships, obtained by the proposed methods, was verified using the nonlinear finite element analysis program *ATENA*. Experimentally investigated beams were modelled employing user defined constitutive model of tensile SFRC. Load-crack width (P-w) relationships were obtained for a given σ_r-w curves, calculated by the Naaman [7], Rilem TC162-TDF [3] and the proposed methods. Simulated P-w curves are compared with the experimental measurements in Fig. 3. As it can be seen, the residual stresses assessed by the Naaman's model very often can be found as conservative. Giving underestimation up to 35%, this method is adequate for the beams with reduced number of fibres crossing the crack plane in critical section (beams *S1-2-F05* and *S2-3-F10*). The method recommended by Rilem overestimates the residual strength of the beams with lower volume of fibres (V_f = 0.5%), giving the prediction error up to 25%. The proposed technique for the determination of residual strength is adequate for all tested beams, giving an error within the margins of 10%.

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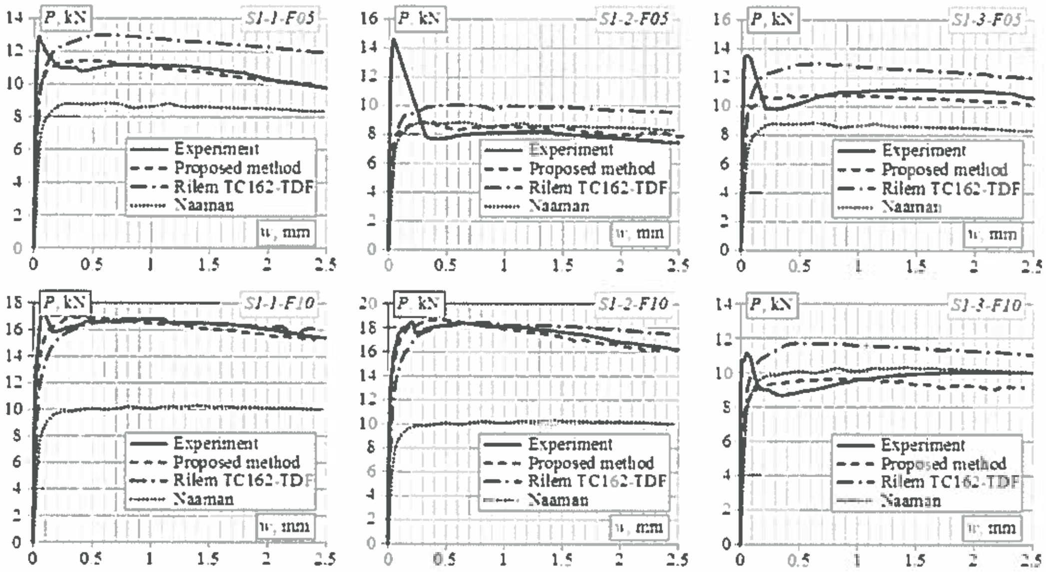

FIGURE 3. Experimental and calculated P-w curves for the SFRC beams with 0.5% (top row) 1.0% (bottom row) fibres content

CONCLUSIONS

The paper deals with experimental and theoretical investigation of the post-cracking behaviour of steel fibre reinforced concrete (SFRC) in tension. Experimental results of six notched beams with fibre contents of 0.5% and 1.0% by volume subjected to three-point loading scheme are presented. The main advantage of the proposed inverse technique for determination of the residual stresses of SFRC is its capability to assess crack opening width of SFRC beams. The numerical analysis has proved adequacy of the inversely derived residual stress-crack opening relations and theirs acceptable for nonlinear finite element analysis as material law of SFRC.

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